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Dynamic attack zone of air-to-air missile after being launched in random wind field

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Flight envelopes;
Numerical calculation procedures;
Simulation analysis

Abstract A new concept is presented for air-to-air missile which is dynamic attack zone after being launched in random wind field. This new concept can be used to obtain the 4-dimensional (4-D) information regarding the dynamic envelope of an air-to-air missile at any flight time aimed at different flight targets considering influences of random wind, in the situation of flight fighters cooperated with missiles fighting against each other. Based on an air-to-air missile model, some typical cases of dynamic attack zone after being launched in random wind field were numerically simulated. Compared with the simulation results of traditional dynamic envelope, the properties of dynamic attack zone after being launched are as follows. The 4-D dynamic attack zone after being launched is inside traditional maximum dynamic envelope, but its forane boundary is usually not inside traditional no-escape dynamic envelope; Traditional dynamic attack zone can just be reliably used at launch time, while dynamic envelope after being launched can be reliably and accurately used during any flight antagonism time. Traditional envelope is a special case of dynamic envelope after being launched when the dynamic envelope is calculated at the launch time; the dynamic envelope after being launched can be influenced by the random wind field.

1. Introduction

The purpose of this paper is to present a new concept for air-to-air missile, dynamic attack zone after being launched in random wind field. This concept can be used in the air combats, because usually many flight fighters are attacked by a lot of air-to-air missiles when flight fighters fight against each other, and sometimes each fighter is attacked by hostile multiple missiles,[1–3] which are shown in Figs. 1 and 2, where h, x and z are respectively the altitude, longitude and latitude distance positions. To improve aggressive effectiveness of the air-to-air

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missiles in the capriccioso situation and unknown situation in near future of the air combat, there are too many problems to be solved, with the two problems listed as follows:

(1) The flight vehicles being attacked by missiles need to know the attack zones of each air-to-air missile at that moment in real time online. When cluster air-to-air missiles attack multitudinous coordinated aerial assault fighters, generally each air-to-air missile locks on and attacks a maneuvering flight target. In the anfractuous 4-dimensional (4-D) flight counter air, a target fighter maybe attacked by several air-to-air missiles. At this moment, this flight target vehicle can maneuver to break through interceptions and get rid of the assaults from all air-to-air missiles which can attack that target, as well as accomplish flight missions. Regarding all these air-to-air missiles, what is the attack zone information of each missile?
(2) Each air-to-air missile requires knowing the attack zones in allusion to different flight targets with their current flight statuses and some characteristics. In the situation of flight fighters fighting against each other, when two air-to-air missiles attack the same flight target, if the flight target has been destroyed by one of them, the other missile can attack the other maneuvering flight targets. Then, which maneuvering target should be attacked by this redundant air-to-air missile? This redundant missile can attack the flight targets which are inside the attack zone of the missile at this flight moment. So this redundant missile needs to know the attack zone regarding flight target statuses at this flight moment in real time online.

When cluster air-to-air missiles attack multitudinous flight targets, generally each missile locks on a maneuvering target

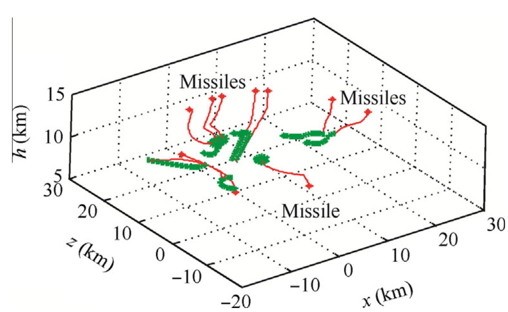


Fig. 1 A group of air-to-air missiles attack multitudinous flight targets.

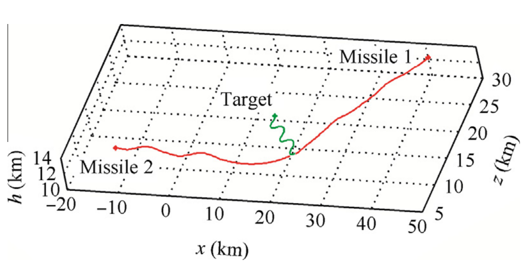


Fig. 2 Two missiles attacking one flight target at the same time.

and attacks the locked target until the missile gets or loses the target; but in the rough-and-tumble aerial warfare, the relative flight statuses including the relative positions and velocities between all missiles and each flight target are changing. Sometimes the attacked targets for all missiles should be redistributed, because this redistribution can make the missiles get better attack results.

Currently the dynamic envelope of air-to-air missiles or dynamic launch zone (DLZ) is just calculated at the launch time by the onboard computers in flight carrier of the missiles. According to the published papers, the attack zones of air-to-air missiles have been researched basically as the following aspects: (1) The high accuracy calculation regarding all kinds of dynamic attack zones for air-to-air missiles, such as the maximum envelope[4,5] and non-escape envelope,[6–8] as well as the launch envelopes for air-to-air missiles;[9,10] the envelope databases are obtained by this calculation for all possible initial conditions of missile and target at the launch time. (2) The approximated calculation of dynamic envelope or DLZ is in real time online, but the approximated DLZ is inside the acceptable error of curve fitting of air-to-air missile attack zones.[11–14] (3) The influences or sensitivity on attack zone boundaries or launch envelope because of all possible stochastic factors, such as the changes of air-to-air missiles launching conditions,[15] random wind field,[16] the errors of missile model,[17,18] etc. But currently there is not any paper in related to the dynamic attack zone of air-to-air missiles after being launched in random wind field. If the concept and flight numerical simulation are researched regarding the dynamic attack zone in random wind of air-to-air missiles after being launched, it is very important for aerial warfare based on flight fighters and missiles. The dynamic attack zones of air-to-air missiles after being launched in random wind are researched and analyzed in detail.

2. Problem formulation on dynamic attack zone after being launched in random wind field

Generally, the traditional dynamic attack zone of air-to-air missile is calculated according to the reference point when the aircraft is launching this missile. Based on the flight states including different entry angles of the flight target, the attack zone boundary can be calculated.

For the air-to-air missiles which can attack flight targets in omnidirectional directions, the attack zone is a continuous envelope which includes the reference point;[4] for the frontal attack air-to-air missile, the attack zone envelope is made up of inner, lateral and outer boundaries. To make a difference between the traditional dynamic attack zone and dynamic attack zone after being launched, the traditional dynamic attack zone is also called dynamic attack zone at launch time.

Definition 1. Dynamic attack zone of air-to-air missile after being launched in random wind field: an air-to-air missile was launched and flied along a trajectory to intercept a flight target, at the flight time t_s in this intercept process, based on the missile flight states including flight velocity and position vector, dynamic attack zone for this missile to attack the other flight targets. The reference point of this dynamic attack zone is the missile flight position at the flight time t_s. The dynamic attack zone after being launched in random wind field is described as

$$\begin{cases} R_{\max} = R_{\max}(V, \gamma, \psi_v, h, x, z, n_{\max}, q_T, N_1, N_2, ..., h_T, V_T, n_T, \xi)|_{t=t_s} \\ R_{\min} = R_{\min}(V, \gamma, \psi_v, h, x, z, n_{\max}, q_T, N_1, N_2, ..., h_T, V_T, n_T, \xi)|_{t=t_s} \\ L_{\max} = L_{\max}(V, \gamma, \psi_v, h, x, z, n_{\max}, q_T, N_1, N_2, ..., h_T, V_T, n_T, \xi)|_{t=t_s} \\ D_{\max} = D_{\max}(V, \gamma, \psi_v, h, x, z, n_{\max}, q_T, N_1, N_2, ..., h_T, V_T, n_T, \xi)|_{t=t_s} \end{cases} \tag{1}$$

where $R_{\max}$ and $R_{\min}$ are respectively the forane boundary and vicinity boundary of the attack zone after being launched at flight time t_s; $L_{\max}$ and $D_{\max}$ are respectively the left border and right border of this attack zone after being launched; $R_{\max}$, $R_{\min}$, $L_{\max}$ and $D_{\max}$ form the whole dynamic attack zone after being launched. $(V, \gamma, \psi_v)|_{t=t_s}$ are velocity, path angle and heading angle of air-to-air missile at flight time t_s; $(h_T, V_T, q_T)|_{t=t_s}$ are respectively the flight height, velocity and the entry angle of the flight target at flight time t_s; $n_{\max}$ and n_T are respectively maximum available maneuver overloads of air-to-air missiles and the flight target; ξ is random wind field; N_1, N_2, ... are other restrictions, such as the work statuses of missile aerodynamic characteristics, propulsion systems, other subsystems and all kinds of random errors and disturbances, maneuver types of flight targets and so on.

3. Numerical algorithm for dynamic attack zone in random wind field after being launched

3.1. Mathematic model of air-to-air missile

The 6-DOF differential motion equations of air-to-air missile are as follows:

$$\dot{\boldsymbol{x}} = f[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{P}_M, \boldsymbol{x}_T(t), t] \tag{2}$$

where flight status $\boldsymbol{x}(t) = [V, \gamma, \psi_v, h, x, z, \omega_x, \omega_y, \omega_z, \Gamma, \varphi, \phi, \alpha, \beta, \sigma]$, with ω_x, ω_y and ω_z the rotation rates of Euler attitude angles, Γ, φ and ϕ the Euler attitude angles, α, β and σ the attack angle, sideslip angle and bank angle; flight control $\boldsymbol{u}(t) = [\delta_x, \delta_y, \delta_z] = [k_{1x}\sigma,\ k_{1y}a_z,\ k_{1z}a_y\ ]$, with δ_x, δ_y and δ_z are rudder angles, k_{1x}, k_{1y} and k_{1z} are coefficients of rudder angles, a_y and a_z the command accelerations of missile; the missile parameters $\boldsymbol{P}_M = [c_y^\alpha,\ c_z^\beta,\ c_y^{\delta_z},\ c_z^{\delta_y},\ \overline{X}pz,\ m_z^\alpha,\ m_y^\beta,\ m_x^\sigma,\ m_x^{\delta_x},\ m_y^{\delta_y},\ m_x^{\overline{\omega}_x},\ m_y^{\overline{\omega}_y},\ m_z^{\overline{\omega}_z},\ m_y^{\overline{w}x},\ m_x^{\overline{w}y},\ I_x,\ I_y,\ I_z,\ I_{xy}, I_{yz}, I_{zx},\ P,\ m,\ S]$, with c_y^α, c_z^β, $c_y^{\delta_z}$ and $c_z^{\delta_y}$ the partial derivative coefficients of aerodynamic forces on the missile, $\overline{X}pz$ the distance between the gravity center and pressure center, m_z^α, m_y^β, m_x^σ, $m_x^{\delta_x}$, $m_y^{\delta_y}$, $m_x^{\overline{\omega}_x}$, $m_y^{\overline{\omega}_y}$, $m_z^{\overline{\omega}_z}$, $m_y^{\overline{w}x}$ and $m_x^{\overline{w}y}$ the partial derivative coefficients of aerodynamic moments, I_x, I_y, I_z, I_{xy}, I_{yz} and I_{zx} the rotational inertia parameters; flight status of target $\boldsymbol{x}_T(t) = [V_T, \gamma_T, \psi_{vT},\ h_T, x_T, z_T]$, where the subscript "T" means the flight target in this paper; S is the reference area of the missile body. The engine thrust P and missile mass m are calculated by

$$P = \begin{cases} F & t \leqslant t_{E\_f} \\ 0 & t > t_{E\_f} \end{cases} \tag{3}$$

$$m = \begin{cases} m_0 - \int_0^t \dfrac{F}{I_{sp}} dt & t \leqslant t_{E\_f} \\ m_0 - m_f & t > t_{E\_f} \end{cases} \tag{4}$$

where m_0 is initial mass of missile; m_f is fuel mass of missile engine; I_{sp} is specific impulse; t_{E_f} is maximum time of the rock engine.

3.2. Missile guidance law

Air-to-air missile uses proportional navigation law of three-dimensional space. In proportional navigation law, in speed coordinates, the command acceleration of air-to-air missile is

$$\begin{cases} a_y = -k_1|\dot{r}|\dot{q}_y \\ a_z = -k_2|\dot{r}|\dot{q}_z \end{cases} \tag{5}$$

where k_1 and k_2 are proportional coefficients; $\dot{r}$ is varying rate of the distance between the target and missile; $\dot{q}_y$ and $\dot{q}_z$ are rotational angle rates of air-to-air missile seeker in speed coordinate system y axis and z axis.

3.3. Target mathematic model

The 3-D differential motion equations of flight target are

$$\begin{cases} \dot{V}_T = a_{Tx}, \quad \dot{\gamma}_T = \dfrac{a_{Ty}}{V_T}, \quad \dot{\psi}_{vT} = \dfrac{a_{Tz}}{V_T \cos \gamma_T} \\ \dot{x}_T = V_T \cos \gamma_T \cos \psi_{vT}, \quad \dot{h}_T = V_T \sin \gamma_T \\ \dot{z}_T = -V_T \cos \gamma_T \sin \psi_{vT} \end{cases} \tag{6}$$

where a_T is maneuver acceleration of target in Earth Cartesian coordinate system. Through setting the function of target flying acceleration (a_{Tx}, a_{Ty}, a_{Tz}) with the change of time, any different types of maneuvering penetration flight trajectories (t, h_T, x_T, z_T) are obtained.

3.4. Random wind field model

$$\boldsymbol{\xi} = [V_{Wx}, V_{Wy}, V_{Wz}, a_{Wx}, a_{Wy}, a_{Wz}]|_{t=t_s} \tag{7}$$

where V_{Wx}, V_{Wy} and V_{Wz} are the component velocities of random wind field in Earth coordinate system; a_{Wx}, a_{Wy} and a_{Wz} are the component accelerations of random wind field in Earth coordinate system. The random wind field model is described in Refs.[19,20] The average velocity of random wind field and wind shear which vary with the height from 0 km to 100 km is shown in Fig. 3, where $\overline{V}_W$ is the average velocity of wind and a_W is the wind shear of wind field.

3.5. Calculations of both dynamic attack zone after being launched and dynamic attack zone at launch time

To calculate envelopes with high accuracy and non-real time online, based on the mathematic models Eqs. (1)–(7) of both the air-to-air missile, the flight target and the random wind model, the dynamic envelope or DLZ of an air-to-air missile at the launch time can be calculated by dichotomy algorithm shown in Figs. 4 and 5 which are presented as follows.

Step 1. Set flight vehicle models and parameters, as well as the initial flight conditions for missile and target.

(1) Missile model and parameters: thruster, aerodynamic, mass properties, missile guidance law and maximum maneuver acceleration, etc.
(2) Missile initial flight status: $(V, \gamma, \psi_v,\ h, x, z,\ \omega_x, \omega_y, \omega_z,\ \Gamma, \varphi, \phi,\ \alpha, \beta, \sigma)|_{t=0 \text{ or } t=t_s}$; the other subsystem information: engine work status and mass properties $(P, m, I_x, I_y, I_z, I_{xy}, I_{yz}, I_{zx})|_{t=t_s}$; missile-target line of sight (LOS) angles $\gamma_{MT}|_{t=t_s}$ and $\psi_{MT}|_{t=t_s}$.

(3) Target parameters: the maneuver type, maximum maneuver acceleration, maximum and minimum flight velocities, etc.
(4) Target initial condition: $(x_T, y_T, z_T, V_{Tx}, V_{Ty}, V_{Tz}, a_{Tx}, a_{Ty}, a_{Tz})|_{t=0 \text{ or } t=t_s}$, where V_{Tx}, V_{Ty} and V_{Tz} are component velocities of target in Earth coordinate system. Set the initial distance $R_{T,Lost,New}$ between missile and target to be enough large and then the missile cannot meet the target; set the initial distance $R_{T,Got,New}$ to be enough smaller and then the missile can get the target.
(5) Wind model: $(V_{Wx}, V_{Wy}, V_{Wz}, a_{Wx}, a_{Wy}, a_{Wz})|_{t=t_s}$. Set the parameters of random wind model.

Step 2. Reset target initial position $R_{T,N}$ by

$$R_{T,N} = 0.5(R_{T,Lost,New} + R_{T,Got,New}) \tag{8}$$

$$(x_T, h_T, z_T)|_{t=t_0} = (x_{T,N}, h_{T,N}, z_{T,N}) \tag{9}$$

$$\begin{cases} x_{T,N} = R_{T,N} \cos \gamma_{MT} \cos \psi_{MT} \\ h_{T,N} = R_{T,N} \sin \gamma_{MT} \\ z_{T,N} = R_{T,N} \cos \gamma_{MT} \sin \psi_{MT} \end{cases} \tag{10}$$

where γ_{MT} and ψ_{MT} are LOS angles which are respectively in plumb and horizontal planes.
Step 3. Flight numerical simulation by Eqs. (2)–(7) for missile and target to obtain miss distance r_{MT}.
Step 4. Increase or decrease $R_{T,N+1}$ by dichotomy algorithm.

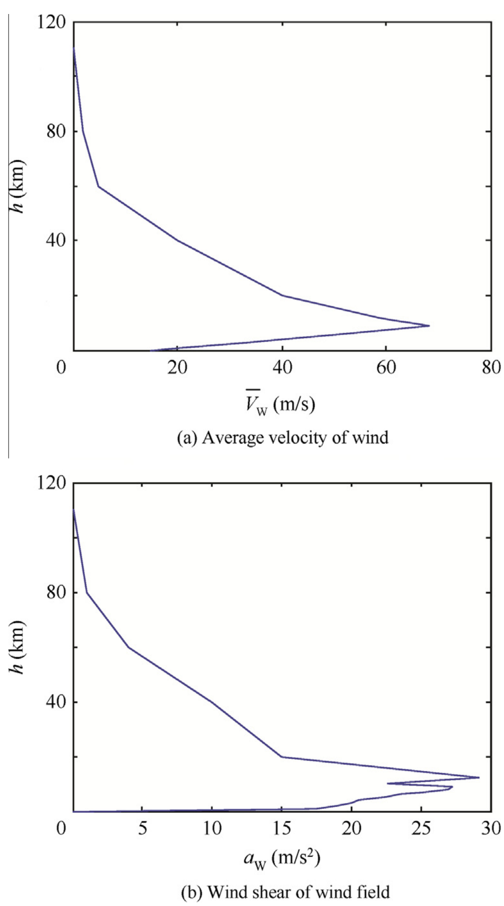


Fig. 3 Random wind field model.

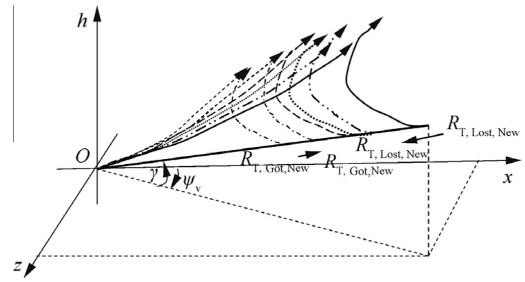


Fig. 4 Iterated process for calculating exact points of dynamic envelope by dichotomy algorithm.

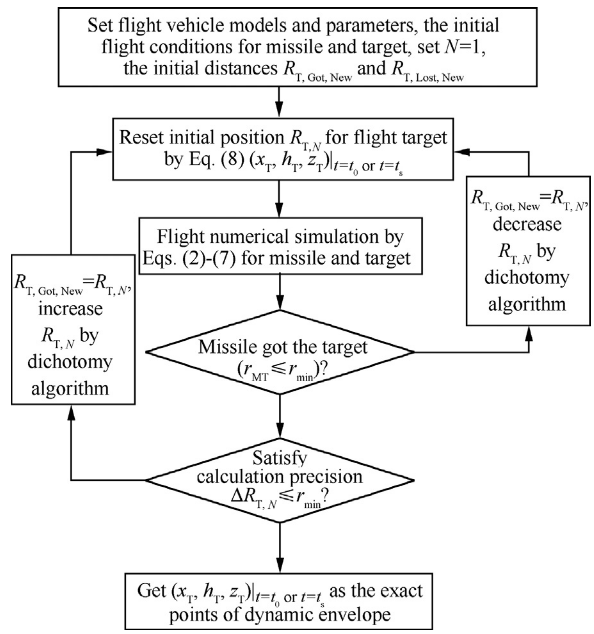


Fig. 5 Calculation flowchart for both dynamic attack zones after being launched and traditional dynamic envelope.

(1) If missile lose the flight target, i.e., $r_{MT} > r_{min}$, with r_{min} the acceptable miss distance, $R_{T,Lost,New} = R_{T,N}$, decrease $R_{T,N}$ by dichotomy algorithm Eq. (8), then go to Step 2.
(2) Otherwise, if $r_{MT} \leqslant$S r_{min}, check the calculation precision

$$\Delta R_{T,N} = |R_{T,N} - R_{T,Got,New}| \leqslant r_{min} \tag{11}$$

where $\Delta R_{T,N}$ is the calculation precision of dynamic envelope or dynamic attack zone after being launched. And then,
(1) If Eq. (11) exists, go to Step 5.
(2) Otherwise, if $\Delta R_{T,N} > r_{min}$, then $R_{T,Got,New} = R_{T,N}$; increase $R_{T,N}$ by dichotomy algorithm Eq. (8) and go to Step 2.

Step 5. Obtain the target initial condition $(x_T, y_T, z_T)|_{t=t_0}$ by Eq. (9) as the boundaries R_{max}, R_{min}, L_{max} and D_{max} in Eq. (1) for dynamic envelope or dynamic attack zone after being launched.

The iterated process for calculating the exact points of dynamic envelope by dichotomy algorithm can be seen in Fig. 4 and the calculation flowchart for both dynamic attack zone after being launched and traditional dynamic envelope is shown in Fig. 5.

4. Numerical simulation results

The calculation conditions for the 3-D traditional envelope and dynamic attack zone in random wind field after being launched are as follows:

(1) The initial flight speed of both the target and the air-to-air missile is 300 m/s.
(2) The available maximum maneuver overload of the target $n_T = 9g$ and the available maximum maneuver overload of the air-to-air missile $n_{max} = 25g$.
(3) Set the initial flight parameters of missile and target and the maneuver type of target.
(4) Missile engine thrust $P = 7000$ N, the maximum time of the rock engine $t_{E\_f} = 10$ s, specific impulse $I_{sp} = 237.89$ s.
(5) Set the type of random wind field (including wind velocities and directions).

Four numerical simulation cases are no-escape envelope and the relative dynamic attack zones after being launched with and without influences of random wind field, as well as the maximum envelope and the relative dynamic attack zone after being launched with and without influences of random wind field. Those numerical simulation results are presented as follows.

Case 1. Traditional no-escape envelope of rear attack, dynamic attack zones of rear attack after being launched 10 s and 30 s without regard to the influences of random wind field.

To calculate the dynamic attack zone of rear attack after being launched 10 s, a lot of flight numerical simulations were done which chose the target initial flight height from 5 km to 12 km after missile being launched 10 s later. Fig. 6 shows the flight trajectories of both missile and target and parameters of missiles when target initial flight height is 7 km after being launched 10 s later, as an example. In Fig. 6, a_c means the guidance command acceleration of missile.

In Fig. 7, the missile envelope presented as "·" in red color is the traditional envelope which is calculated at the time $t = 0$ s when the air-to-air missile is launched; the missile envelope presented as "o" in blue color is the dynamic attack zone after being launched 10 s later; the ballistic curve is the flight trajectory for the missile to fly in the first 10 s starting from the initial launch status. For calculating the traditional envelope at flight time $t = 0$ s, the flight targets fly without any maneuver and the missiles pursue the targets all along from the targets' rear. For calculating the dynamic attack zone after being launched at flight time $t = 10$ s, the targets fly to the missiles at first, then the trajectories of targets are controlled by U-turn maneuver and later the missiles pursue the targets all along by rear attack. It is the same to calculate dynamic attack zone after being launched at flight time $t = 30$ s and the results can be seen in Fig. 8. The dynamic attack zone after being launched 10 s and 30 s is not completely inside the no-escape zone at $t = 0$ s. The envelope of the dynamic attack zone after being launched is not symmetry because of the U-turn maneuvers of targets.

Case 2. Traditional maximum envelope of frontal attack, dynamic attack zones after being launched 10 s and 30 s of frontal attack without regard to the influences of random wind field.

For calculating the traditional envelope at flight time $t = 0$ s, the flight targets fly without any maneuver and the missiles pursue the targets all along by frontal attack. For calculating the dynamic attack zone after being launched at flight time $t = 10$ s, the targets fly away from the missiles and the trajectories of targets are controlled by U-turn maneuver and then no maneuver, so later the missiles pursue the targets all along by frontal attack. It is the same for calculating the dynamic attack zone after being launched at flight time $t = 30$ s. The simulation results are shown in Figs. 9 and 10. The dynamic attack zone after being launched 10 s and 30 s is completely inside the maximum attack zone at $t = 0$ s. The envelope of the dynamic attack zone after being launched is not symmetric because of the U-turn maneuvers of targets.

Case 3. Dynamic attack zone after being launched 10 s of frontal attack and 30 s of rear attack considering the random wind field.

Set the random wind field model, whose velocity is about half the average value of the standard wind field model and direction is along the positive direction of x axis. This means missile flies downwind. The simulation results are shown in Figs. 11 and 12.

By making a contrast between Figs. 9 and 11, when missiles fly downwind whose velocity is about 30 m/s and attacking head on the targets which maneuver by U-turn, the dynamic attack zone after being launched 10 s has changed. The forane boundary of the dynamic attack zone increases from 70 km to 74 km and the change rate is 5.71%. The lateral boundary of the dynamic attack zone moves towards the positive direction of z axis for 5 km and the change rate is 7.69%. By making a contrast between Figs. 8 and 12, when missiles fly downwind whose velocity is about 30 m/s and attack after the targets which maneuver by U-turn, the dynamic attack zone after being launched 30 s has changed. The forane boundary of the dynamic attack zone increases from 34 km to 36 km and the change rate is 5.88%.The lateral boundary of the dynamic attack zone moves towards the positive direction of z axis for 1 km and the change rate is 4.35%.

The forane boundary of the dynamic attack zone after being launched has changed because the wind direction is along the positive direction of x axis. Due to the targets maneuver by U-turn, the lateral boundary is also influenced by the wind field.

If there are contravention results between the traditional envelope and dynamic attack zone after being launched, the result of dynamic attack zone after being launched is correct and the traditional one is incorrect, because they selected different reference points, different moving target flight situation and so on, then the missile flight situation and inner system status are different from those at launch time.

Obviously, traditional envelope is just correct for the flight situations of missile and target at time $t = 0$ s and it cannot be used for the flight situations of missile and target at later time $t > 0$ s; the longer the flight countermove time, the more uncertain and inexact the traditional envelope is and maybe the more mistakes; the dynamic attack zone after being launched can supply the exact and reliable envelope information according to the flight situations of missile and target at

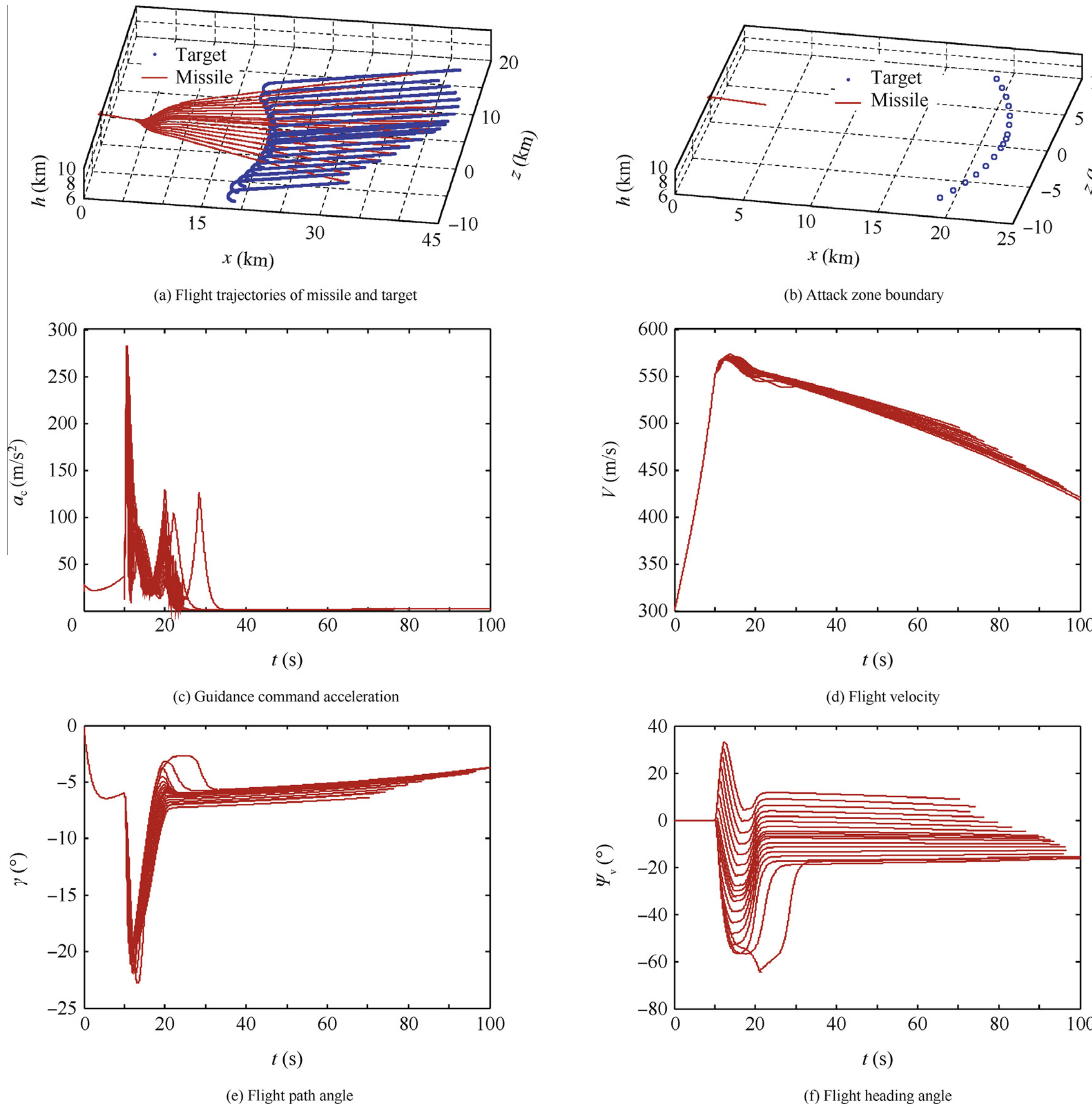


Fig. 6 Flight parameters of missile and target when target initial flight height is 7 km after missile being launched 10 s later.

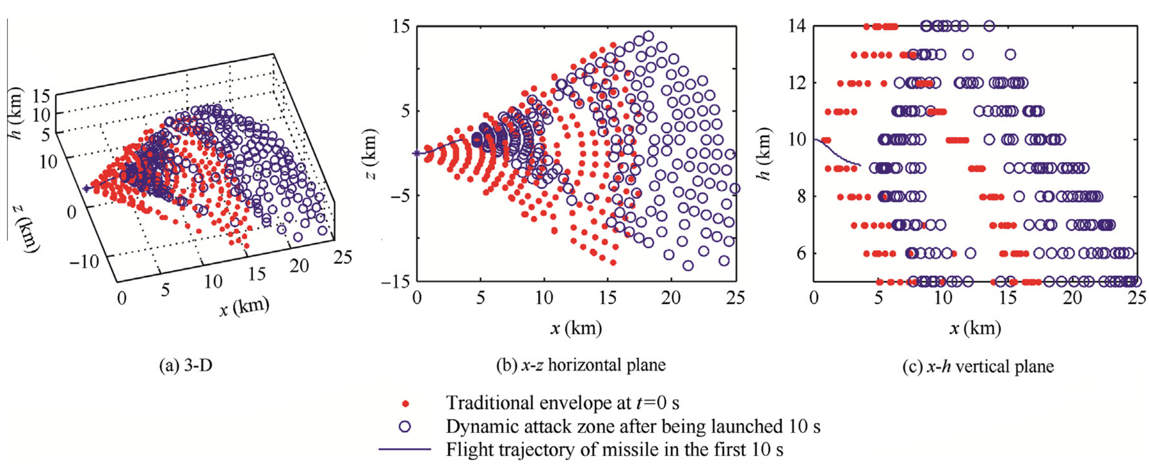


Fig. 7 Corporation of traditional dynamic attack zone and dynamic attack zone after being launched 10 s later.

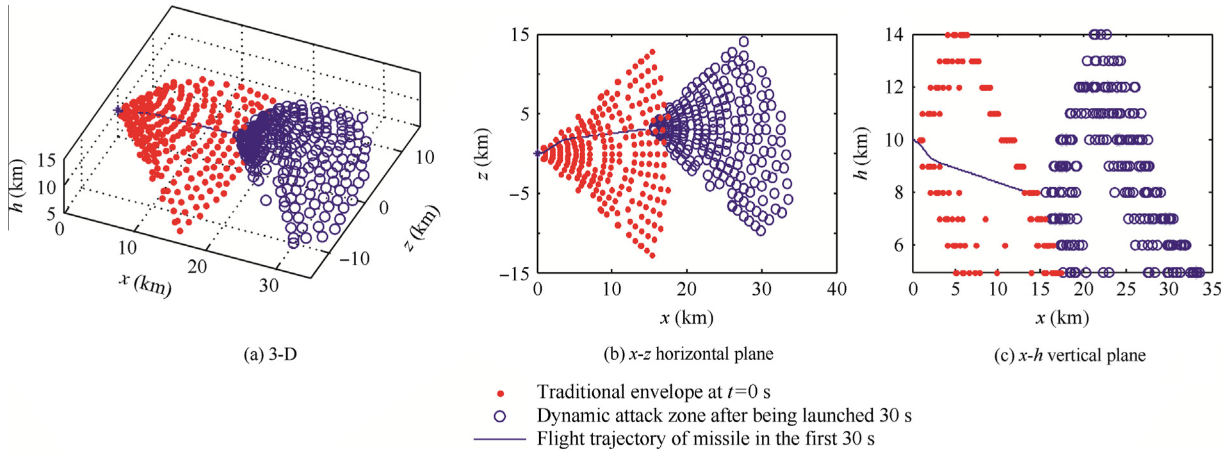


Fig. 8 Corporation of traditional dynamic attack zone and dynamic attack zone after being launched 30 s later.

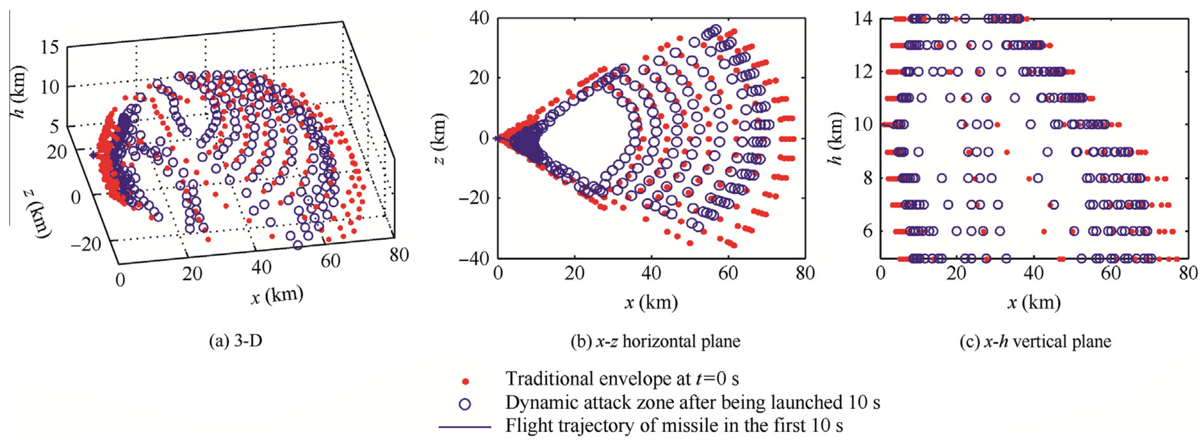


Fig. 9 Corporation of traditional dynamic attack zone and dynamic attack zone after being launched 10 s later.

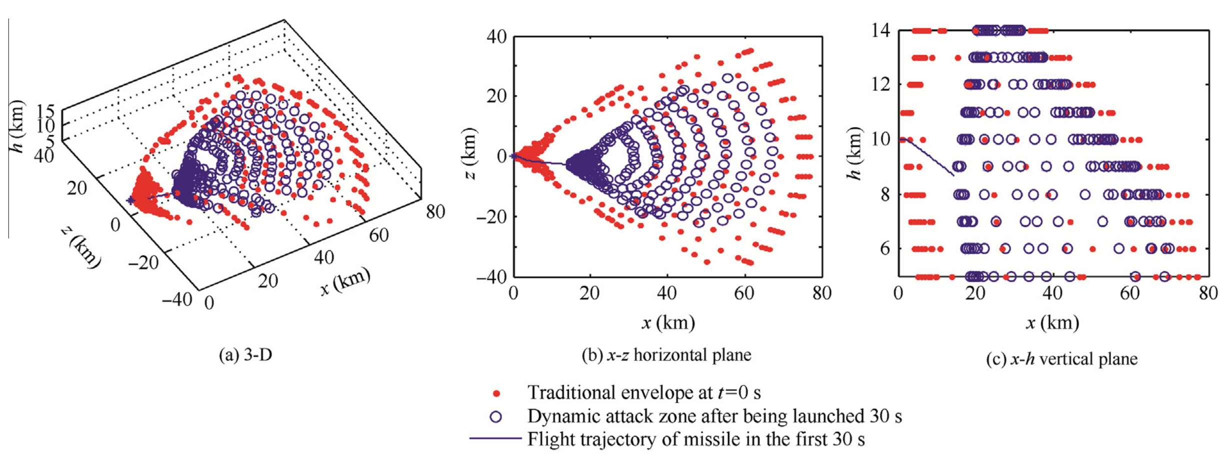


Fig. 10 Corporation of traditional dynamic attack zone and dynamic attack zone after being launched 30 s later.

that current time $t = t_s > 0$ s, based on the missile flight situation $\boldsymbol{x}(t_s)$ and the inner sub-system statuses $\boldsymbol{P}_M(t_s)$, as well as the target flight situation $\boldsymbol{x}_T(t_s)$; when it is necessary to take the random wind field into account, it is more effective to use the dynamic attack zone after being launched.

Case 4. One application of dynamic attack zone after being launched in random wind field.

Two air-to-air missiles (M1 and M2) which are launched at the same time intercept one target (T1). After 28.48 s later, M1 has succeeded in hitting the T1 while M2 has not. But at this

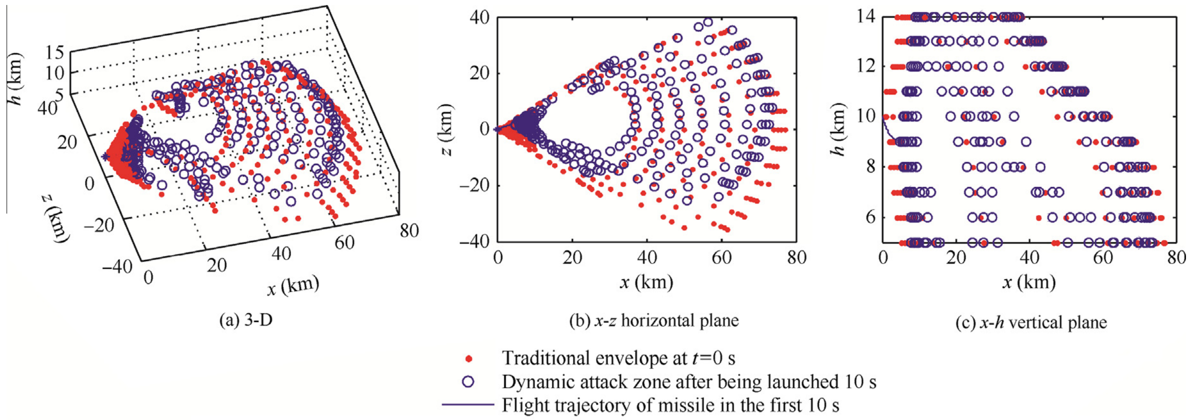


Fig. 11 Corporation of traditional dynamic attack zone and dynamic attack zone after being launched 10 s later.

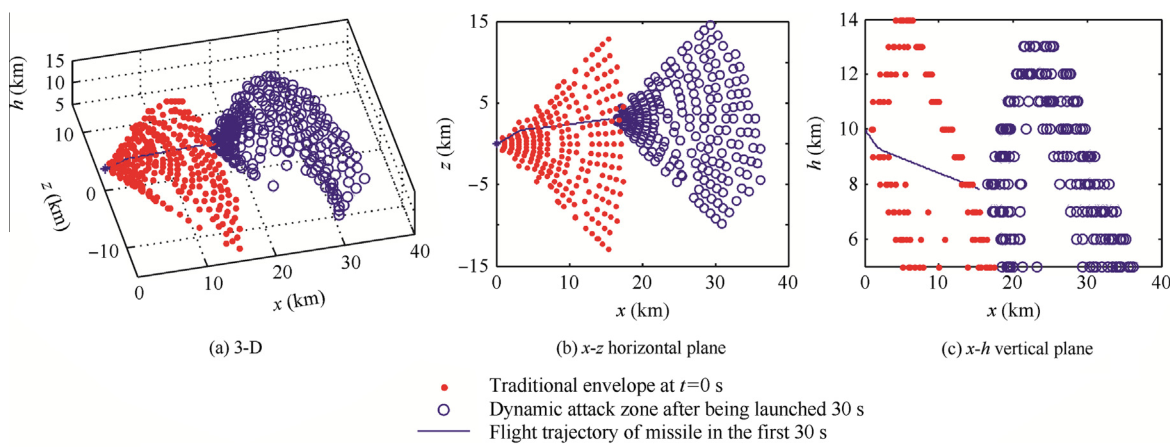


Fig. 12 Corporation of traditional dynamic attack zone and dynamic attack zone after being launched 30 s later.

Table 1 Initial parameters of missiles and targets.

Missile and target	x_0 (km)	h_0 (km)	z_0 (km)	V_0 (m/s)	γ_0 (°)	ψ_{v0} (°)	Maneuver type
M1	0	10	0	300	0	0	
M2	0	10	−5	300	0	0	
T1	13	12	0	300	0	180	U-turn
T2	60	8	5	300	0	180	No

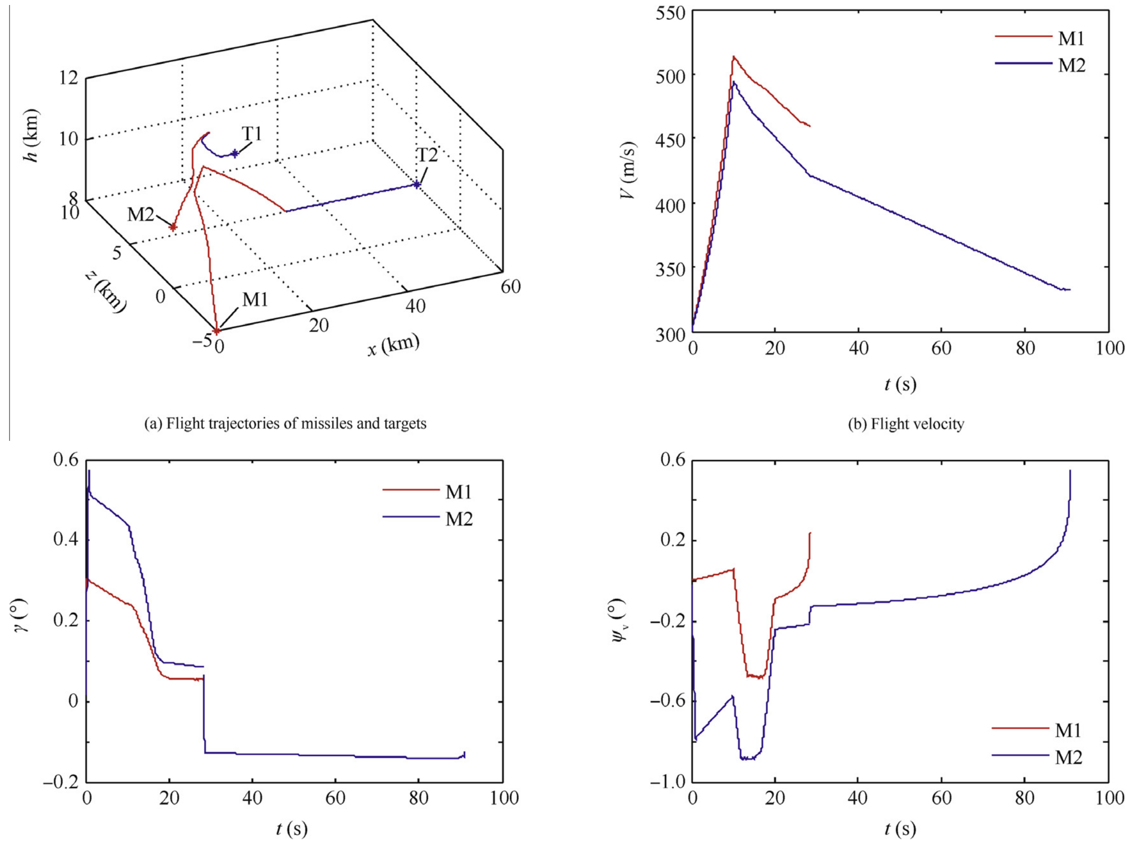
time, the other target (T2) is just in the dynamic attack zone of M2. This means M2 has the ability to hit the T2 after being launched for 28.28 s later. Then M2 will pursuit T2 and hit it finally.

Set the velocity of random wind field to be about half the average value of the standard wind field model and the direction is along the positive direction of z axis. The initial parameters of missiles and targets are shown in Table 1. The subscript "0" means the initial parameters of missile and target and the flight simulation results are shown in Fig. 13.

5. Conclusions

A new concept regarding the air-to-air missile envelope has been presented. This new concept is dynamic attack zone of air-to-air missile after being launched in random wind field. The definition and meaning of dynamic attack zone after being launched were systematically dissertated; numerical simulation algorithm with high accuracy and non-real time online was presented for the dynamic attack zone after being launched; some typical numerical simulation cases were presented regarding dynamic attack zone after being launched based on the same air-to-air missile model; comparisons were made between the dynamic attack zone after being launched and the traditional dynamic attack zone, also comparisons were made between the dynamic attack zone after being launched without considering wind field and the dynamic attack zone after being launched considering wind field.

Based on a great number of flight numerical simulation results, compared with the traditional air-to-air missile envelope which is calculated at the time when the missile is launched, the properties of the dynamic attack zone after being launched are summarized as follows:



(a) Flight trajectories of missiles and targets

(b) Flight velocity

(c) Flight path angle

(d) Flight heading angle

Fig. 13 Flight simulation results of one application of dynamic attack zone after being launched in random wind field.

(1) Traditional maximum dynamic attack zone can always include dynamic attack zone of air-to-air missile after being launched.
(2) Usually dynamic attack zone after being launched is not completely inside the traditional no-escape envelope; it depends on reselection of the target flight state and evasive maneuver type, but definitely inside the traditional maximum envelope.
(3) The fight envelope of dynamic attack zone of air-to-air missile after being launched is different from the one of traditional dynamic attack zone; traditional dynamic attack zone can just be reliably used at launch time, while dynamic attack zone after being launched can be reliably and accurately used during any flight antagonism time.
(4) Traditional envelope is a special case of dynamic envelope after being launched when the new concept envelope is calculated at the launch time.
(5) Random wind field can impact the dynamic envelope of missile to some degree and it is necessary to consider the influences of random wind field in real-war.

If the envelopes after being launched are calculated by real-time online algorithm without very high accuracy, this concept 'dynamic attack zone after being launched in random wind field can be used in both engineering design and air combat.

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